ISTITUTO
DI TECNOLOGIE DELLA
COMUNICAZIONE,
DELL'INFORMAZIONE
E DELLA
PERCEZIONE

PERCRO Perceptual Robotics Laboratory

Scuola Superiore Sant'Anna

Elementi di FEM

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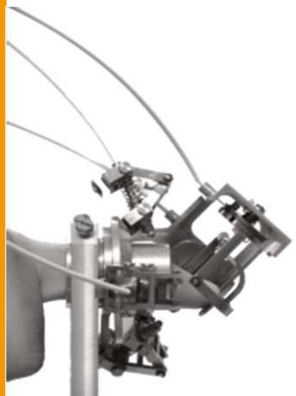
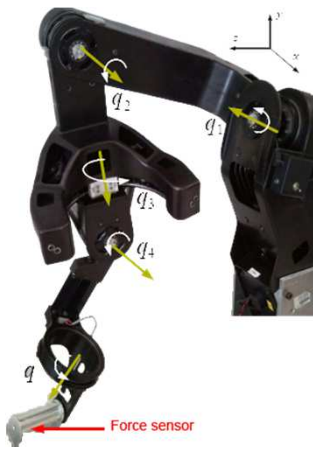
PERCRO, TeCIP Institute, Scuola Superiore Sant'Anna





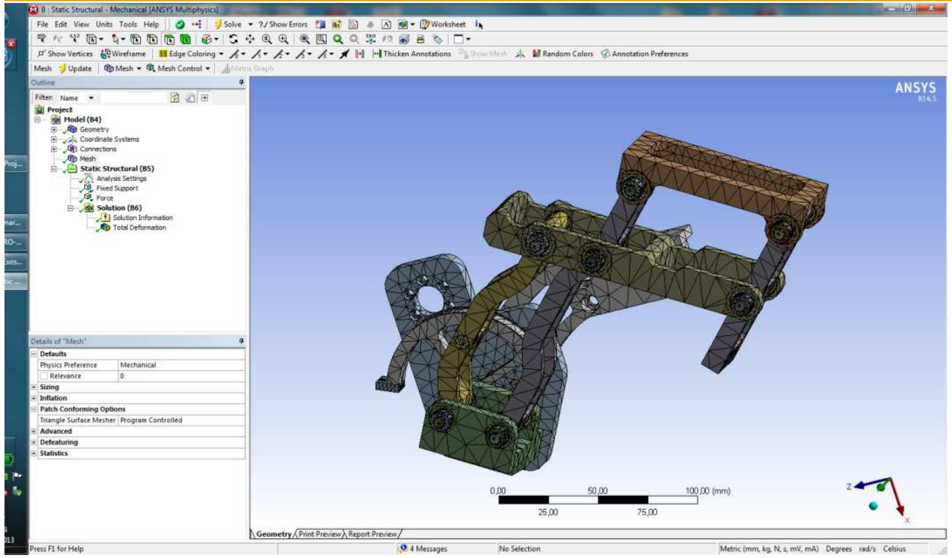
Obiettivi del corso

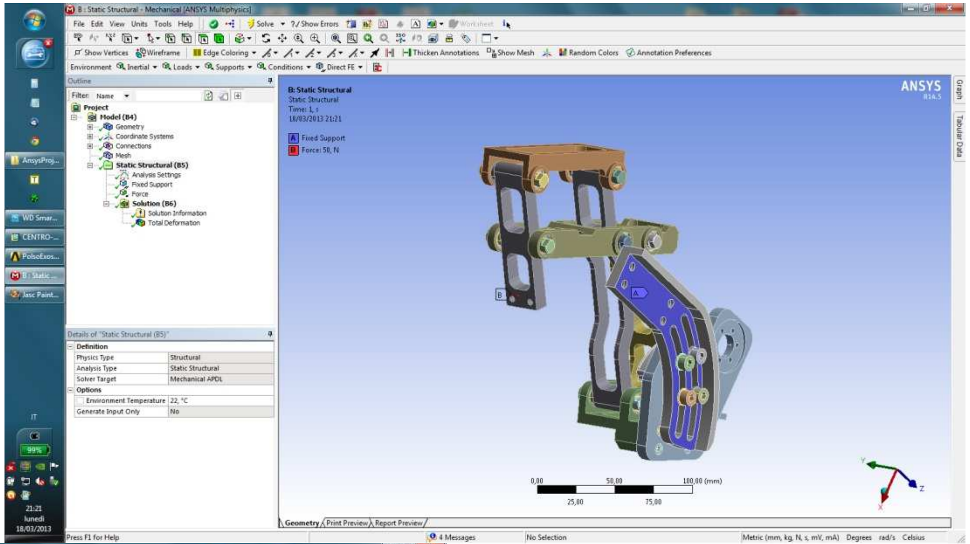
- Fornire degli strumenti operativi con basi teoriche introduttive e principi funzionali di progettazione per la sintesi ed analisi di componenti ed assemblati meccanici
- Pro-E come piattaforma per la modellazione
- Ansys Workbench come piattaforma per lo sviluppo dei nostri esempi

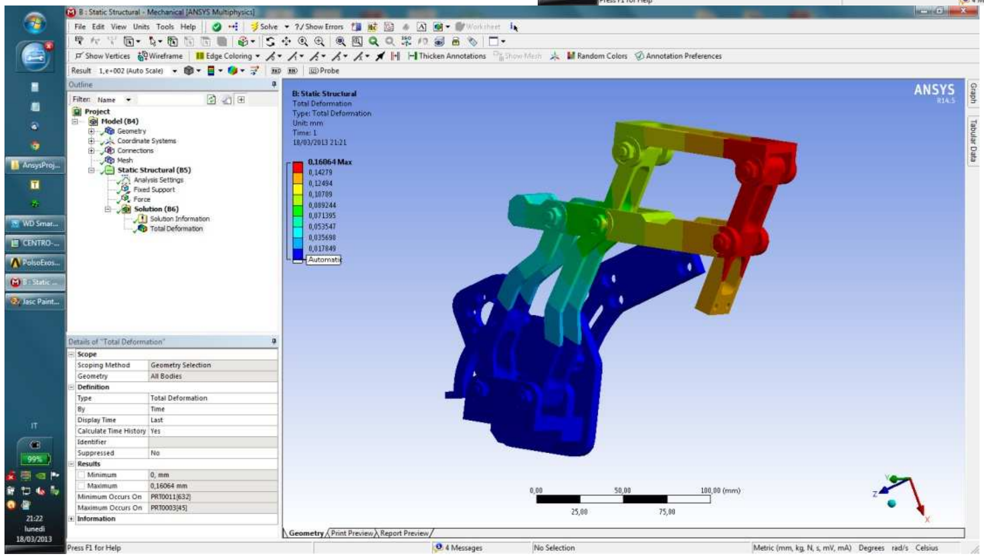
Metodi

- Analisi di case studies notevoli
  - o Confronto tra tecniche ordinarie e tecniche di modellazione basate su simulazione numerica
    - Es soluzioni di sistemi di iperstaticità di grado 1 con ansys e con formulazioni energetiche
    - Iperstaticità di sistemi a n gdl
- 5 postazioni per lo svolgimento di esercitazioni
  - o Main limits
    - 30K nodes allowed as maximum for simulation
    - Maximum 5 licenses running simultaeously

1	Fem introduction and introduction to geometry modeler in Ansys workbench (example of solution of simple cases)
2	Introduction to interface of Workbench & Pro-E (simple examples, including parts only or subassemblies)
3	Design practices (basic principles) and constraint theory
4	Materials (caratteristiche ed utilizzo)
5	isostatica sistema di attuazione
6	Struttura seriale n dof importazione in workbench, analisi rigidezza (rehab exos)
7	Struttura isostatica 3dof (cella esoscheletro maniglia) esempio parallelo
8	Stiffness analysis: Sensori di forza basati su strain gauges, sensore 1 dof e croce di malta
9	modal analysis and structural dynamics (esempi da libro)
10	Adaptive meshing and convergence issues, Contact regions, stress concentration aspects
11	non liner simulations (flexural hinges)







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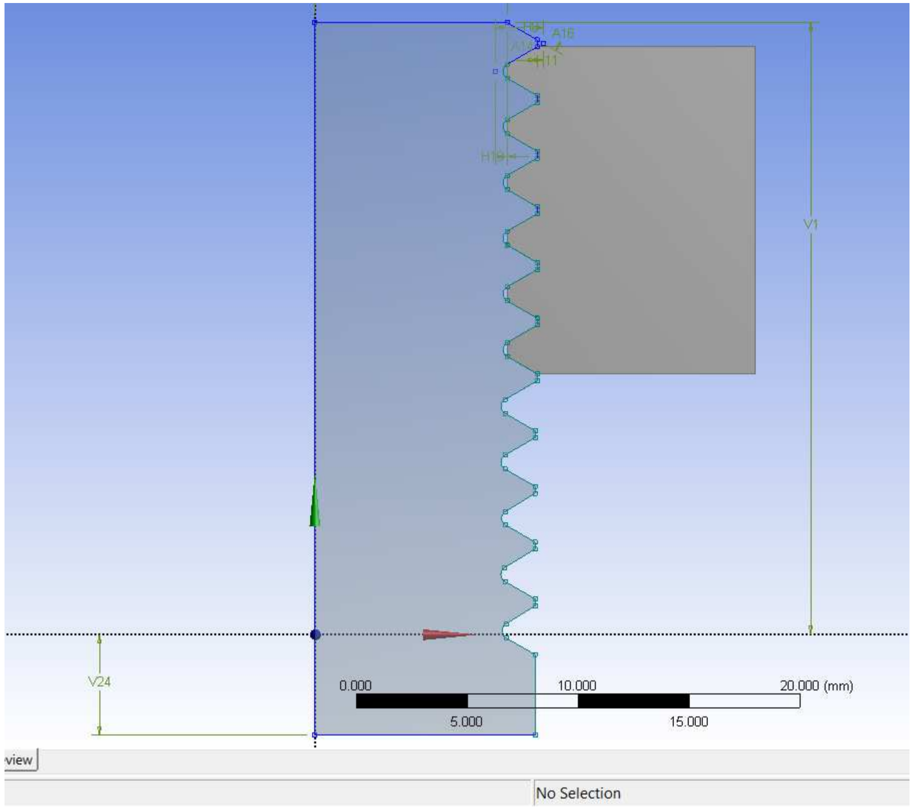
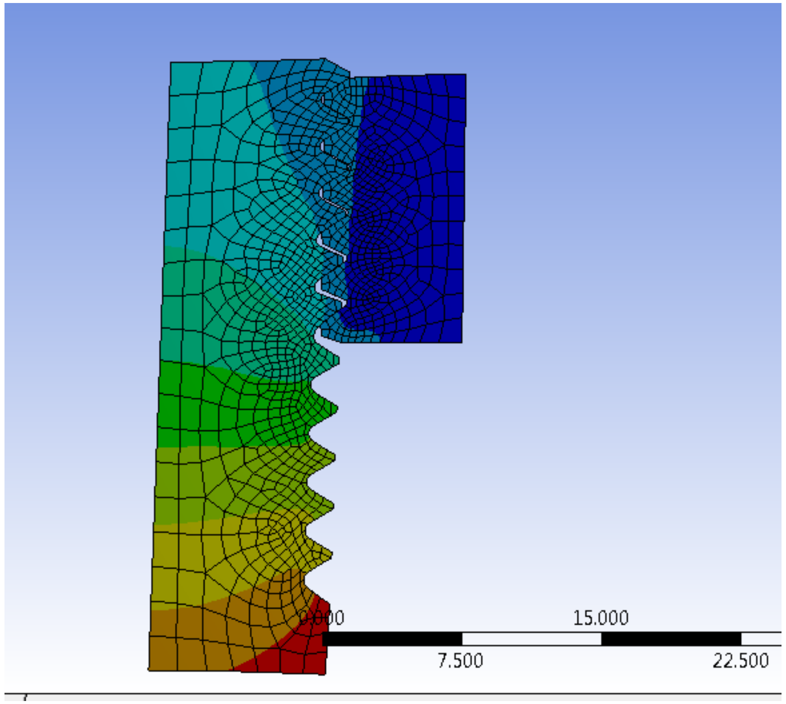
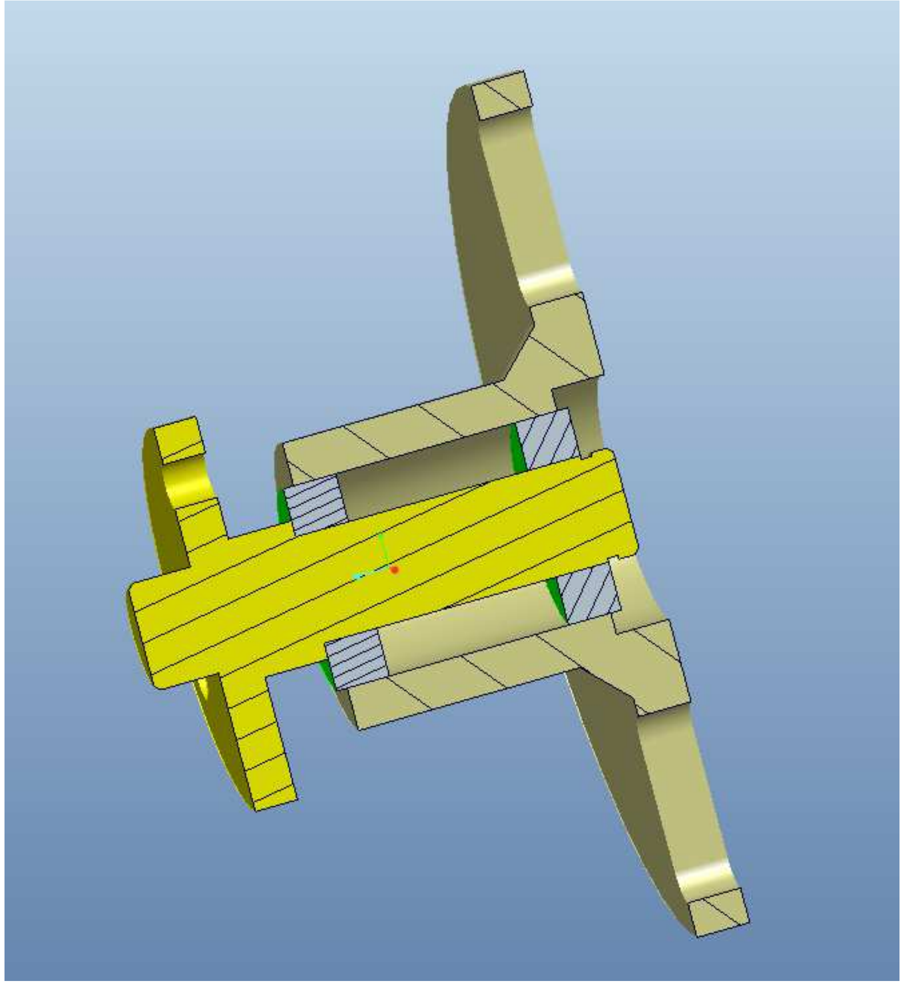
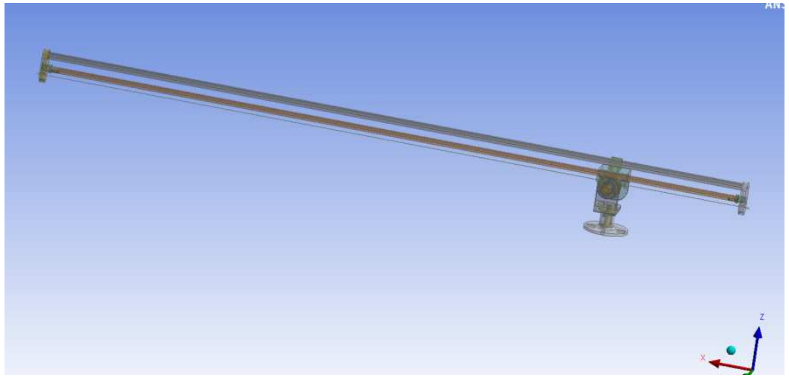

V1
V24
0.000
5.000
10.000
15.000
20.000 (mm)
No Selection

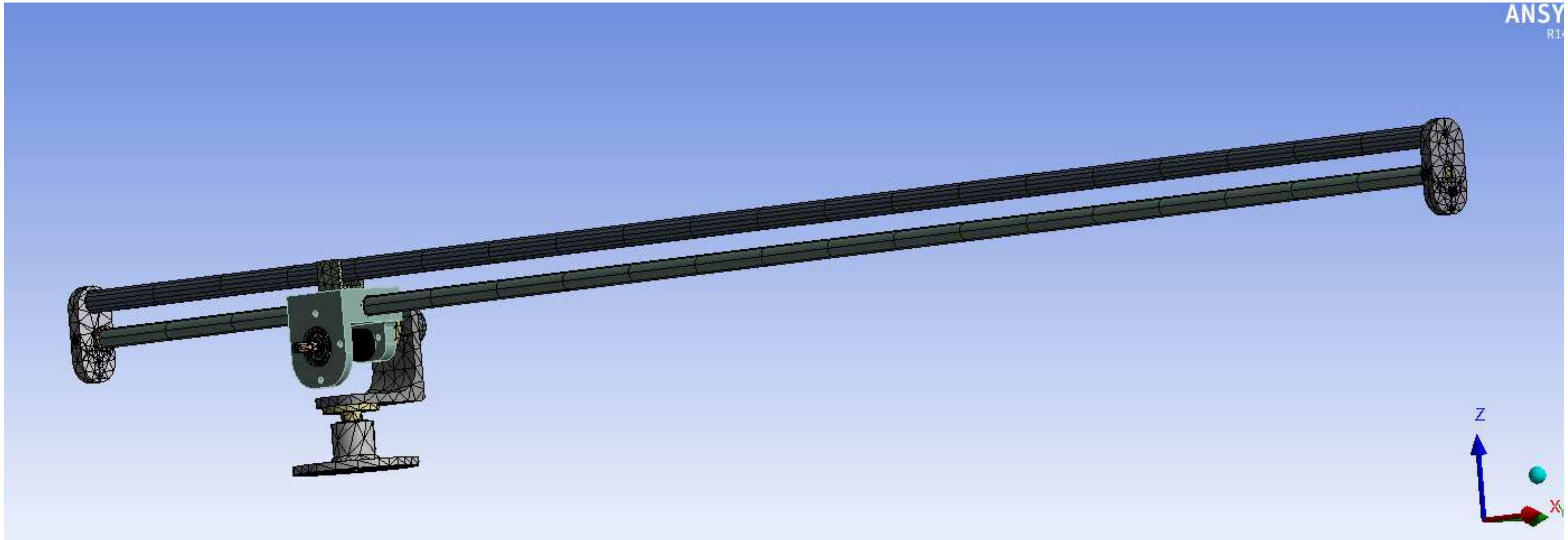

7.500
15.000
22.500

- In questo modo il nostro assemblato è pronto per essere esportato.



- Ecco il modello importato in ansys

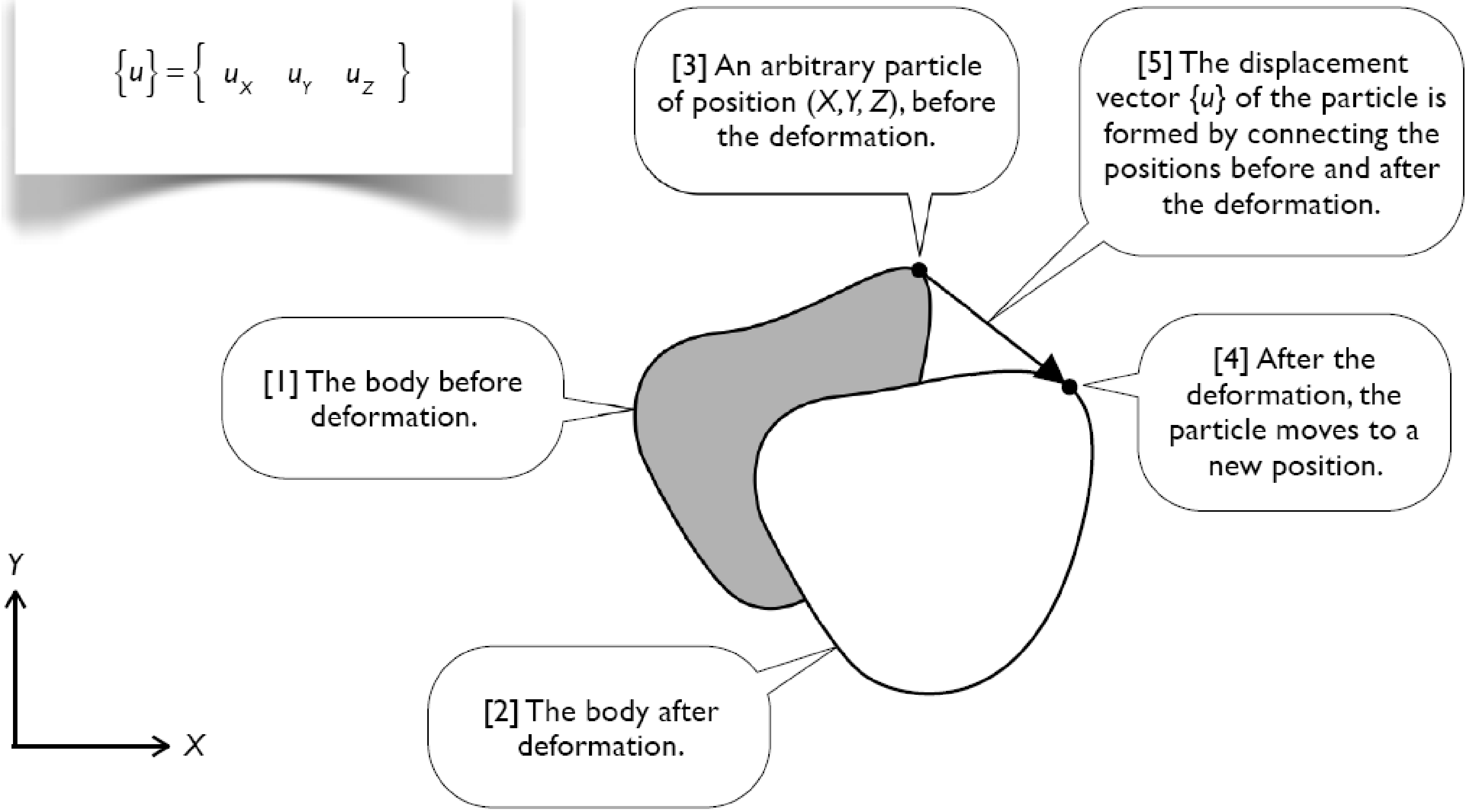




Principi di FEM: una introduzione concettuale

- Displacements

$$\{u\} = \{ \; u_X \quad u_Y \quad u_Z \; \}$$



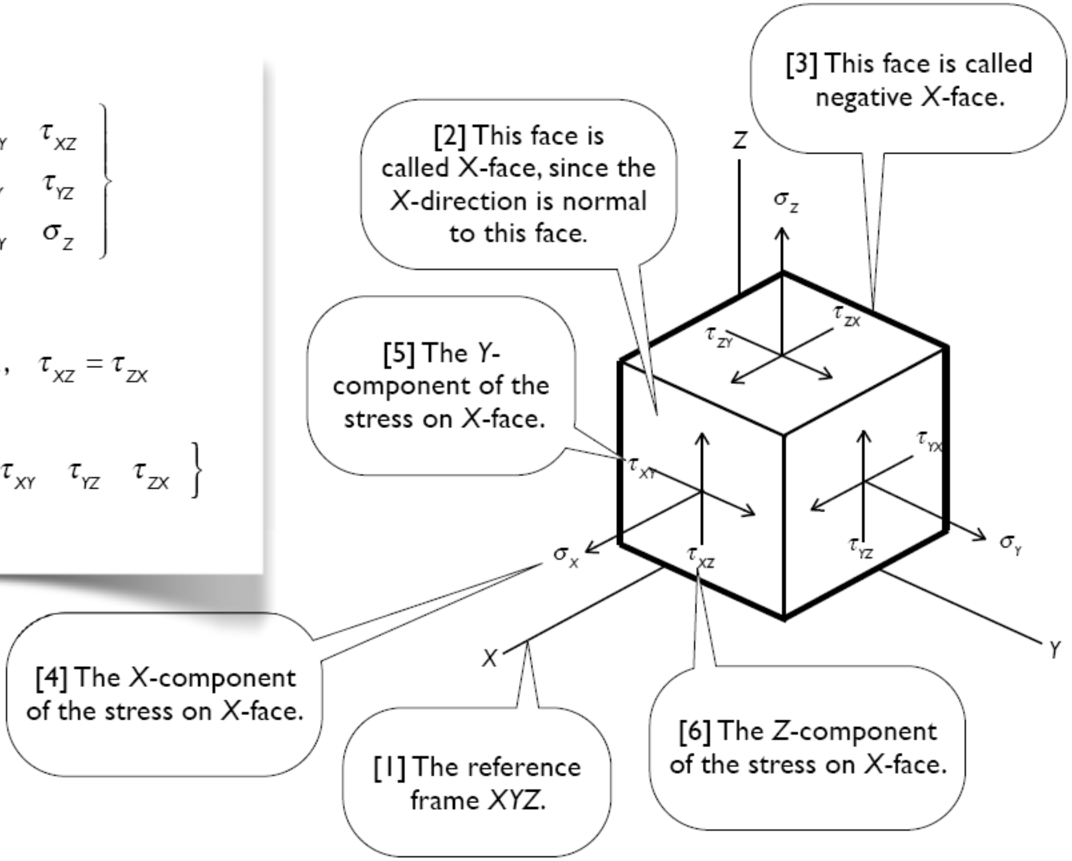
Il tensore delle tensioni

Stresses

$$\{\sigma\} = \begin{Bmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_Y & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_Z \end{Bmatrix}$$

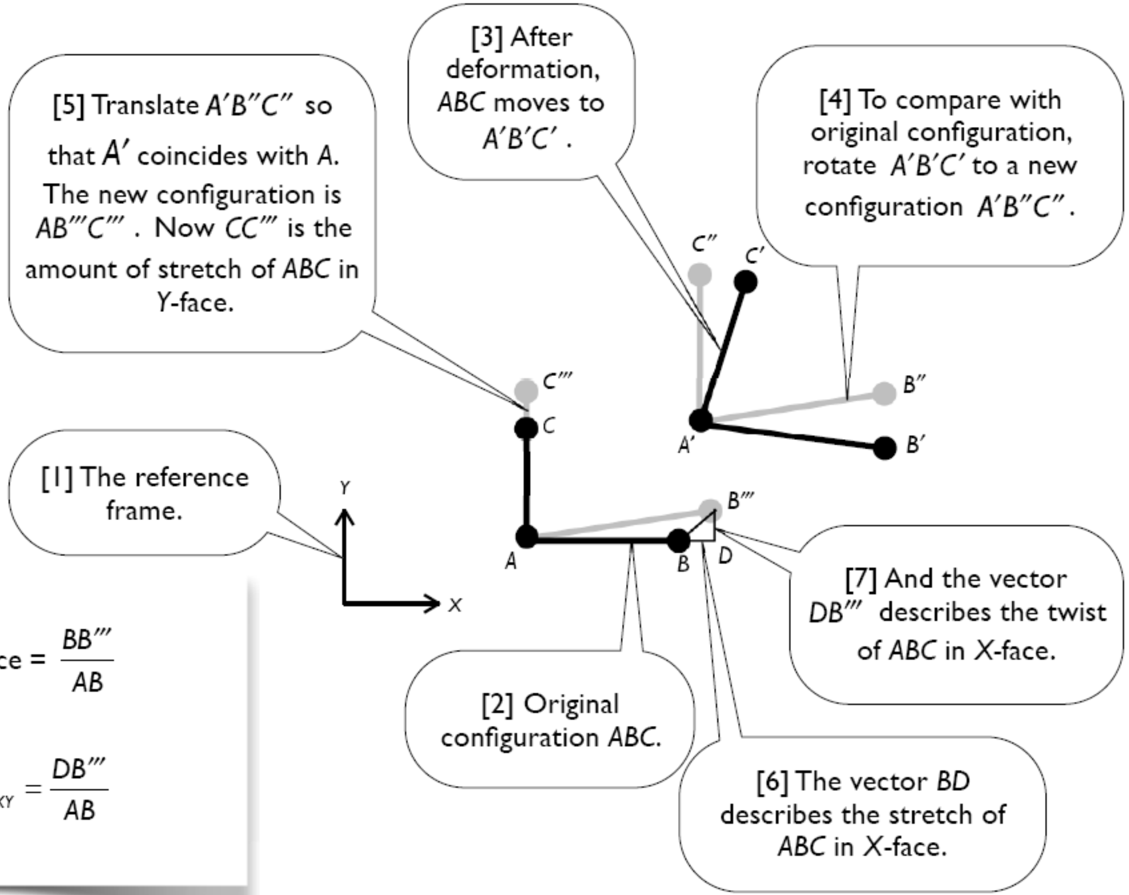
$$\tau_{XY} = \tau_{YX}, \quad \tau_{YZ} = \tau_{ZY}, \quad \tau_{XZ} = \tau_{ZX}$$

$$\{\sigma\} = \{\ \sigma_X \quad \sigma_Y \quad \sigma_Z \quad \tau_{XY} \quad \tau_{YZ} \quad \tau_{ZX}\ \}$$



Deformazioni

Strains



$$\text{Strain on } X\text{-face} = \frac{BB'''}{AB}$$

$$\varepsilon_X = \frac{BD}{AB}, \quad \gamma_{XY} = \frac{DB'''}{AB}$$

Significato fisico delle deformazioni

- Physical meaning of strains:
  - The *normal strain* ε_X is the percentage of stretch of a fiber which lies along X-direction.
  - The *shear strain* γ_{XY} is the angle change (in radian) of two fibers lying on XY-plane and originally forming a right angle.
- We can define other strain components in a similar way.

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_X & \gamma_{XY} & \gamma_{XZ} \\ \gamma_{YX} & \varepsilon_Y & \gamma_{YZ} \\ \gamma_{ZX} & \gamma_{ZY} & \varepsilon_Z \end{Bmatrix}$$

$$\gamma_{XY} = \gamma_{YX}, \quad \gamma_{YZ} = \gamma_{ZY}, \quad \gamma_{XZ} = \gamma_{ZX}$$

$$\{\varepsilon\} = \{ \varepsilon_X \quad \varepsilon_Y \quad \varepsilon_Z \quad \gamma_{XY} \quad \gamma_{YZ} \quad \gamma_{ZX} \}$$

Le equazioni costitutive

$$\{u\} = \{\ u_X \quad u_Y \quad u_Z\ \}$$

$$\{\sigma\} = \{\ \sigma_X \quad \sigma_Y \quad \sigma_Z \quad \tau_{XY} \quad \tau_{YZ} \quad \tau_{ZX}\ \}$$

$$\{\varepsilon\} = \{\ \varepsilon_X \quad \varepsilon_Y \quad \varepsilon_Z \quad \gamma_{XY} \quad \gamma_{YZ} \quad \gamma_{ZX}\ \}$$

Totally 15 quantities

- Equilibrium Equations (3 Equations)
- Strain-Displacement Relations (6 Equations)
- Stress-Strain Relations (6 Equations)

Stress strain relationship Hooke's law

$$\varepsilon_X = \frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E}$$

$$\varepsilon_Y = \frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E} - \nu\frac{\sigma_X}{E}$$

$$\varepsilon_Z = \frac{\sigma_Z}{E} - \nu\frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E}$$

$$\gamma_{XY} = \frac{\tau_{XY}}{G}, \quad \gamma_{YZ} = \frac{\tau_{YZ}}{G}, \quad \gamma_{ZX} = \frac{\tau_{ZX}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

- For *isotropic, linearly elastic* materials, *Young's modulus* (*E*) and *Poisson's ratio* (ν) can be used to fully describe the stress-strain relations.
- The Hooke's law is called a *material model.*
- The Young's modulus and the Poisson's ratio are called the material parameters of the material model.

Se teniamo conto della dilatazione termica

$$\varepsilon_X = \frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E} + \alpha\Delta T$$

$$\varepsilon_Y = \frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E} - \nu\frac{\sigma_X}{E} + \alpha\Delta T$$

$$\varepsilon_Z = \frac{\sigma_Z}{E} - \nu\frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E} + \alpha\Delta T$$

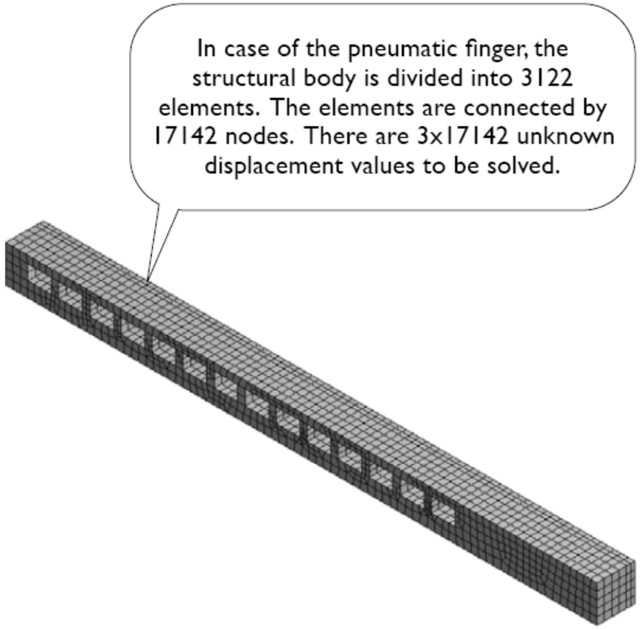
$$\gamma_{XY} = \frac{\tau_{XY}}{G}, \quad \gamma_{YZ} = \frac{\tau_{YZ}}{G}, \quad \gamma_{ZX} = \frac{\tau_{ZX}}{G}$$

- If temperature changes (thermal loads) are involved, the coefficient of thermal expansion, (CTE, α) must be included.
- If inertia forces (e.g., dynamic simulations) are involved, the mass density must be included.

L'idea alla base del metodo agli elementi finiti

- A basic idea of finite element methods is to divide the structural body into small and geometrically simple bodies, called elements, so that equilibrium equations of each element can be written, and all the equilibrium equations are solved simultaneously
- The elements are assumed to be connected by nodes located on the elements' edges and vertices.

- Another idea is to solve unknown discrete values (displacements at the nodes) rather than to solve unknown functions (displacement fields).
- Since the displacement on each node is a vector and has three components (in 3D cases), the number of total unknown quantities to be solved is three times the number of nodes.
- The nodal displacement components are called the degrees of freedom (DOF's) of the structure.



Esempio 1gdl

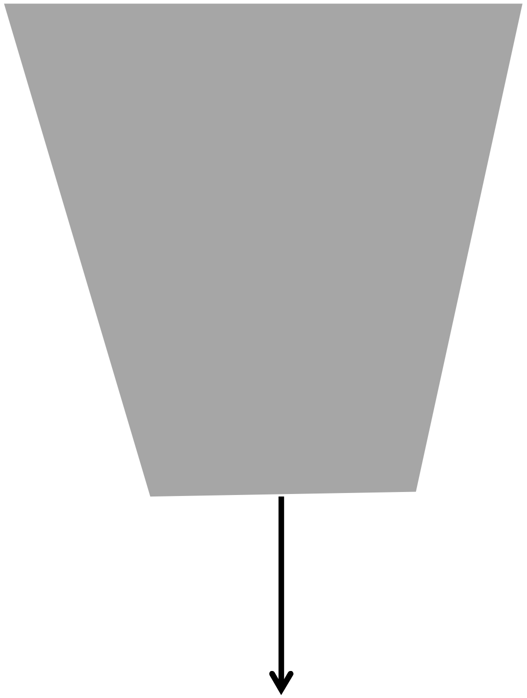
Analisi con bullone

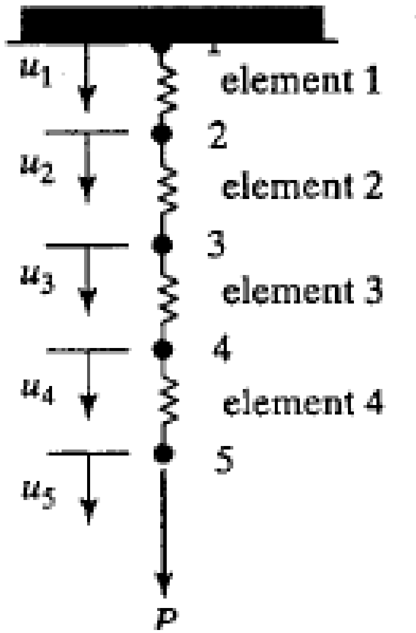
- Dal menu concept selezione surface from sketch
- Quindi subito dopo prima di dare apply selezionare lo sketch dall'albero
- Quindi dare il comando generate
- La sezione deve essere chiusa per poter

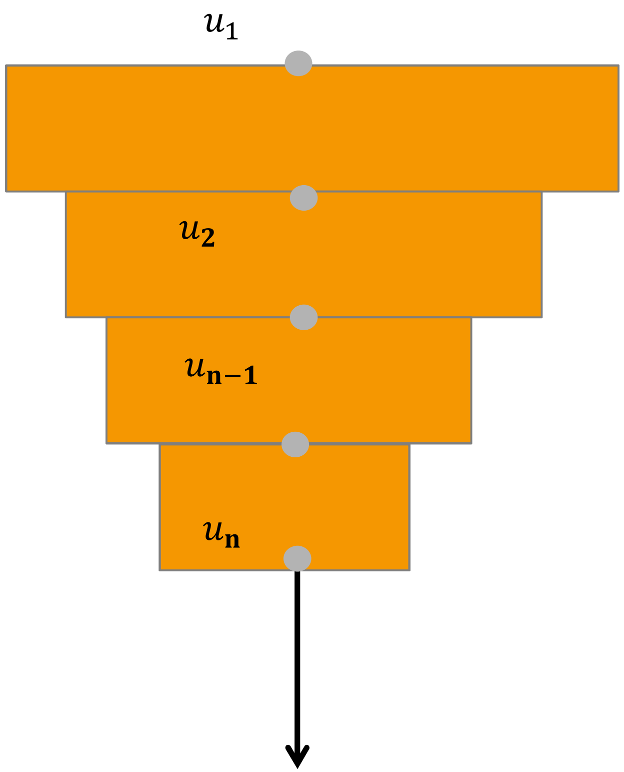
Esempio di analisi numerica FEM

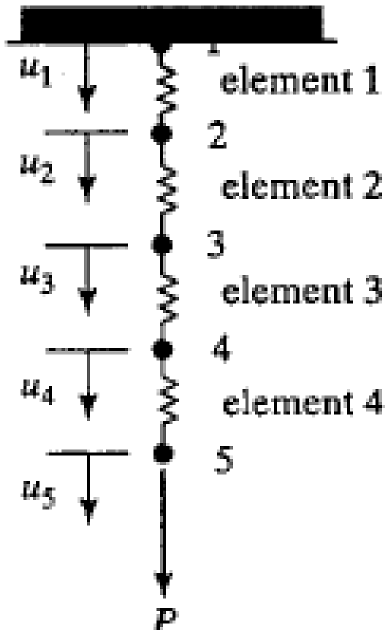
- Supponiamo di analizzare il seguente problema, suddividiamo il problem ain N elementi

$$k = \frac{AE}{l}$$









$$f = k_{eq}(u_{i+1} - u_i) = \frac{A_{avg} E}{\ell}(u_{i+1} - u_i) = \frac{(A_{i+1} + A_i)E}{2\ell}(u_{i+1} - u_1)$$

$$k_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

node 1: $R_1 - k_1(u_2 - u_1) = 0$

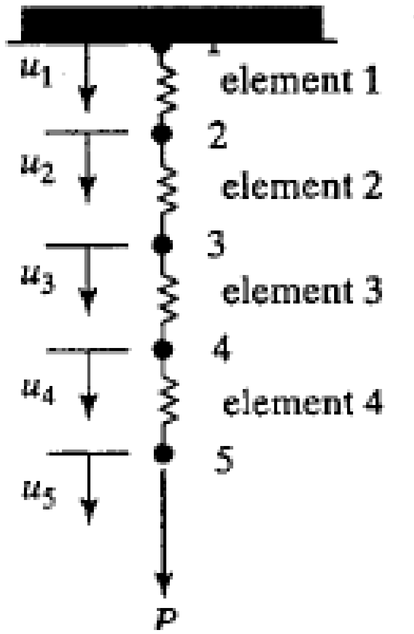
node 2: $k_1(u_2 - u_1) - k_2(u_3 - u_2) = 0$

node 3: $k_2(u_3 - u_2) - k_3(u_4 - u_3) = 0$

node 4: $k_3(u_4 - u_3) - k_4(u_5 - u_4) = 0$

node 5: $k_4(u_5 - u_4) - P = 0$

$$
\begin{aligned}
k_1 u_1 \; - k_1 u_2 \; & = -R_1 \\
-k_1 u_1 + k_1 u_2 + k_2 u_2 - k_2 u_3 \; & = 0 \\
-k_2 u_2 + k_2 u_3 + k_3 u_3 - k_3 u_4 \; & = 0 \\
-k_3 u_3 + k_3 u_4 + k_4 u_4 - k_4 u_5 & = 0 \\
-k_4 u_4 + k_4 u_5 & = P
\end{aligned}
$$



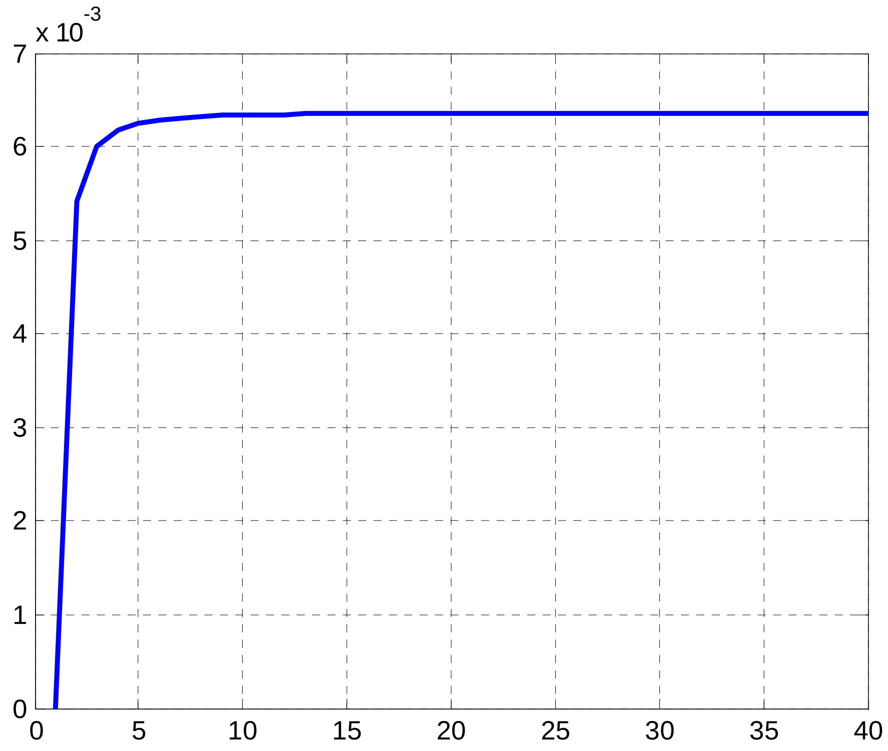
$$
\begin{Bmatrix} -R_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix}
$$

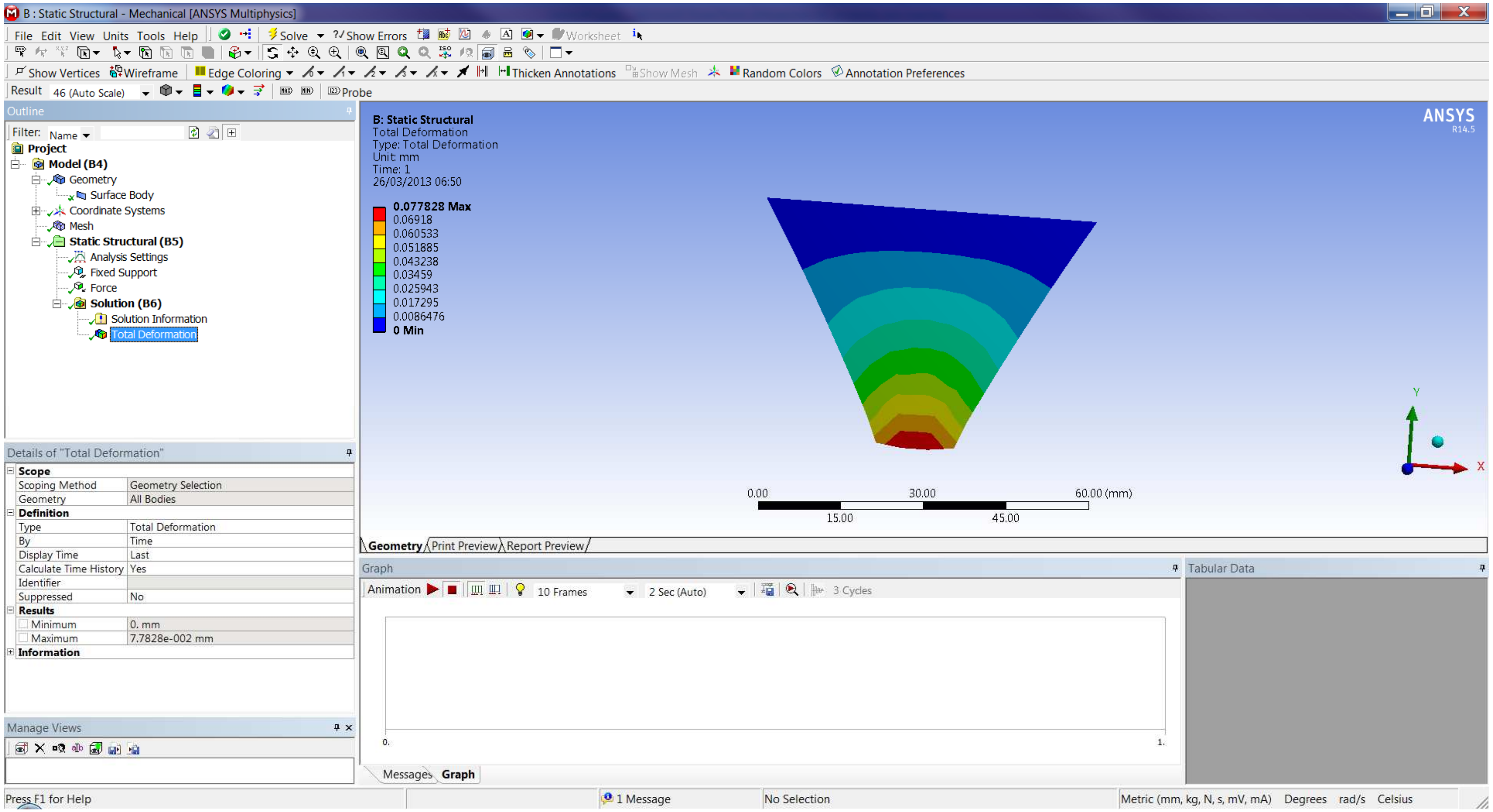
$$\{\mathbf{R}\} = [\mathbf{K}]\{\mathbf{u}\} - \{\mathbf{F}\}$$

- But the first node is constrained to zero, so the first equation should be

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix}$$

Convergency issue





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Esempio 2gdl

Estensione delle funzioni di forma al caso 2D

- Supponiamo di affrontare un problema bidimensionale.
- Dato lo spostamento u=[ux uy uz] di ciascun punto, lo stato di deformazione è definito in modo univoco

$$[\varepsilon]^T = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{zz} & \gamma_{xy} & \gamma_{yz} & \gamma_{xz} \end{bmatrix}$$

Tensore di strain per piccole deformazioni

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Equazioni costitutive

- In caso di materiale di Hook, il tensore delle deformazioni ed il tensore di stress sono collegati da una relazione lineare

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz} \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Dove
E: modulo di Young
v: modulo di Poisson
G: shear modulus

Caso 2D: plain stress

- Nel caso di plain stress, cioè nell'ipotesi che sia sigma_z=0, le equazioni costitutive si riducono alle seguenti:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\boldsymbol{\sigma}\} = [\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}$$

Caso 2D: plain strain

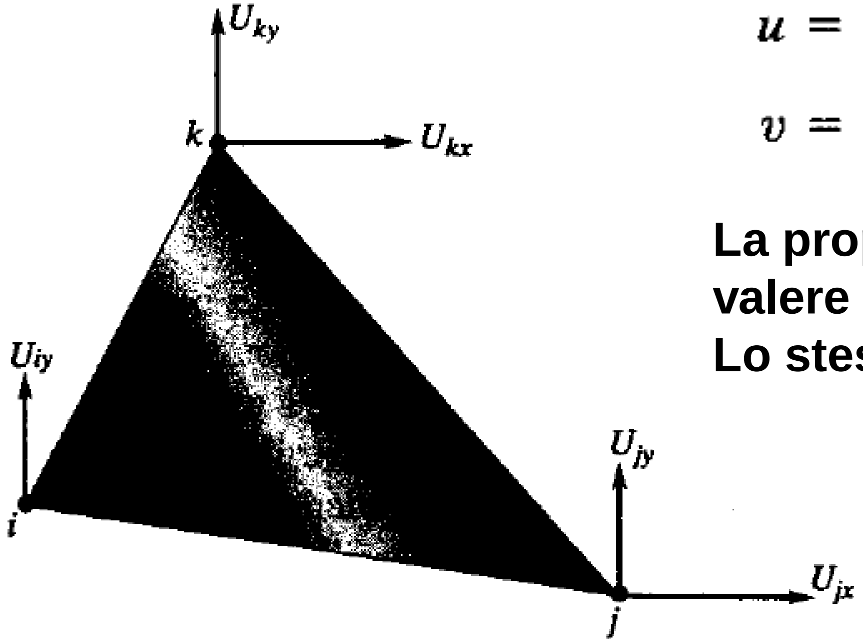
- In caso di plain strain le equazioni diventano:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1}{2}-\nu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\boldsymbol{\sigma}\} = [\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}$$

Funzioni di forma

- Per un elementro triangolare è possibile definire le funzioni di forma



$$u = S_i U_{ix} + S_j U_{jx} + S_k U_{kx}$$

$$v = S_i U_{iy} + S_j U_{jy} + S_k U_{ky}$$

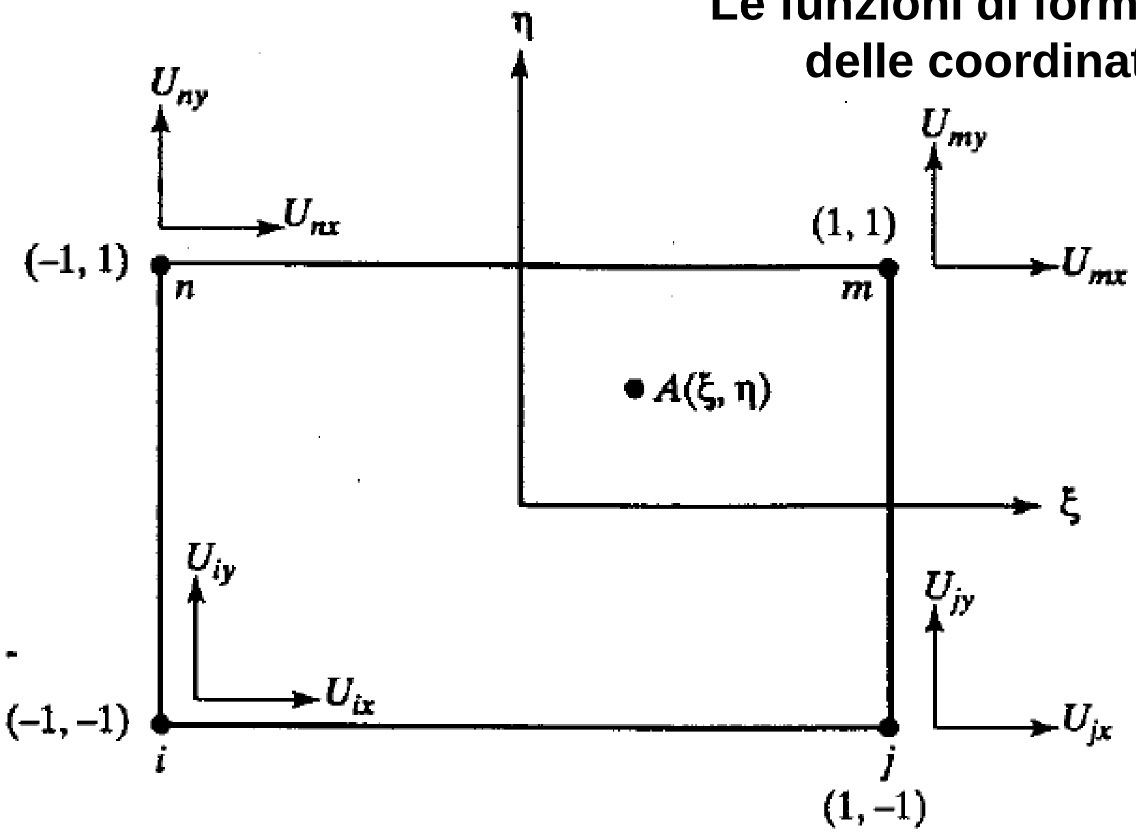
La proprietà delle funzioni di forma è quella di valere Si=1 per il punto i e 0 per I punti j e k
Lo stesso per Sj ed Sk

Caso di un elementro di forma quadrata

- Supponiamo di analizzare un elemento di forma quadrata

Ora le coordinate ξ,μ hanno valore -1, 1

Le funzioni di forma Si(ξ,μ) sono funzione delle coordinate normalizzate ad 1



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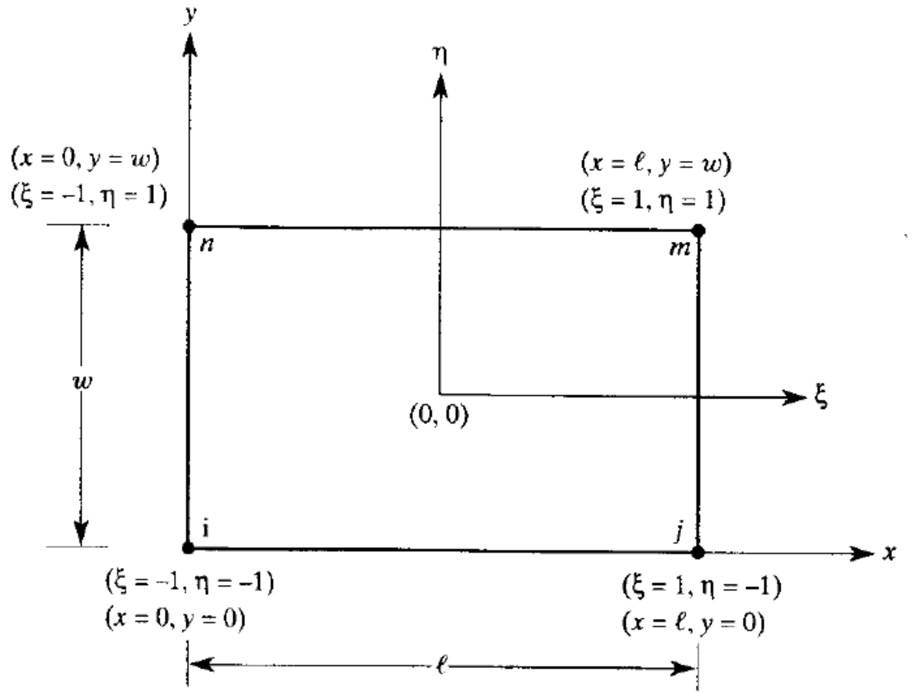
Natural coordinates

- Introduciamo le coordinate naturali e le funzioni di forma per l'elemento quadrilatero



$$\begin{cases} \xi = \dfrac{2x}{\ell} - 1 \\ \eta = \dfrac{2y}{w} - 1, \end{cases}$$

- Le funzioni di forma associati a queste coordinate e all'elemento rettangolare sono chiaramente

$$S_i = \frac{1}{4}(1 - \xi)(1 - \eta)$$

$$S_j = \frac{1}{4}(1 + \xi)(1 - \eta)$$

$$S_m = \frac{1}{4}(1 + \xi)(1 + \eta)$$

$$S_n = \frac{1}{4}(1 - \xi)(1 + \eta)$$

- Funzione di forma ci dice che

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \underbrace{\begin{bmatrix} S_i & 0 & S_j & 0 & S_m & 0 & S_n & 0 \\ 0 & S_i & 0 & S_j & 0 & S_m & 0 & S_n \end{bmatrix}}_{\mathbf{D}} \begin{Bmatrix} U_{ix} \\ U_{iy} \\ U_{jx} \\ U_{jy} \\ U_{mx} \\ U_{my} \\ U_{nx} \\ U_{ny} \end{Bmatrix}$$

Jacobiano

- Definiamo come Jacobiano la matrice che determina il cambio di coordinate

$$\begin{Bmatrix} \dfrac{\partial f(x,y)}{\partial \xi} \\ \dfrac{\partial f(x,y)}{\partial \eta} \end{Bmatrix} = \overbrace{\begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix}} \begin{Bmatrix} \dfrac{\partial f(x,y)}{\partial x} \\ \dfrac{\partial f(x,y)}{\partial y} \end{Bmatrix}$$

$\frac{\partial f}{\partial \xi}$ ↑ **J** ↓ **J**$^{-1}$ $\frac{\partial f}{\partial x}$

$$[\mathbf{J}] = \frac{1}{4}\begin{bmatrix} [-(1-\eta)x_i + (1-\eta)x_j + (1+\eta)x_m - (1+\eta)x_n] & [-(1-\eta)y_i + (1-\eta)y_j + (1+\eta)y_m - (1+\eta)y_n] \\ [-(1-\xi)x_i - (1+\xi)x_j + (1+\xi)x_m + (1-\xi)x_n] & [-(1-\xi)y_i - (1+\xi)y_j + (1+\xi)y_m + (1-\xi)y_n] \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

INVERSA

$$[\mathbf{J}]^{-1} = \frac{1}{J_{11}J_{22} - J_{12}J_{21}} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix}$$

Energia di un elemento

- L'energia di un elemento è data da

$$\Lambda^{(e)} = \frac{1}{2}\int_V \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right) dV$$

$$\downarrow$$

$$\Lambda^{(e)} = \frac{1}{2}\int_V [\boldsymbol{\sigma}]^T\{\boldsymbol{\varepsilon}\}\, dV$$

$$\downarrow$$

$$\Lambda^{(e)} = \frac{1}{2}\int_V \{\boldsymbol{\varepsilon}\}^T[\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}\, dV$$

$$\downarrow$$

$$\Lambda^{(e)} = \frac{1}{2}\int_V \{\boldsymbol{\varepsilon}\}^T[\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}\, dV = \frac{1}{2}(t_e)\int_A \{\boldsymbol{\varepsilon}\}^T[\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}\, dA$$

- Definiamo

$$\{\boldsymbol{\varepsilon}\} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \overbrace{\frac{1}{\det \mathbf{J}} \begin{bmatrix} J_{22} & -J_{12} & 0 & 0 \\ 0 & 0 & -J_{21} & J_{11} \\ -J_{21} & J_{11} & J_{22} & -J_{12} \end{bmatrix}}^{[\mathbf{A}]} \begin{Bmatrix} \dfrac{\partial u}{\partial \xi} \\ \dfrac{\partial u}{\partial \eta} \\ \dfrac{\partial v}{\partial \xi} \\ \dfrac{\partial v}{\partial \eta} \end{Bmatrix}$$

$\dfrac{\partial f}{\partial \xi}$

$\mathbf{J}$ ↑ ↓ $\mathbf{J}^{-1}$

$\dfrac{\partial f}{\partial x}$

$$\{\boldsymbol{\varepsilon}\} = [\mathbf{A}][\mathbf{D}]\{\mathbf{U}\}$$

Questo perché la funzione di forma:

- Funzione di forma ci dice che

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} S_i & 0 & S_j & 0 & S_m & 0 & S_n & 0 \\ 0 & S_i & 0 & S_j & 0 & S_m & 0 & S_n \end{bmatrix} \begin{Bmatrix} U_{ix} \\ U_{iy} \\ U_{jx} \\ U_{jy} \\ U_{mx} \\ U_{my} \\ U_{nx} \\ U_{ny} \end{Bmatrix}$$

Se facciamo la derivata rispetto alle coordinate locali

- Definiamo

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{Bmatrix} = \overbrace{\frac{1}{4}\begin{bmatrix} -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) & 0 \\ -(1-\xi) & 0 & -(1+\xi) & 0 & (1+\xi) & 0 & (1-\xi) & 0 \\ 0 & -(1-\eta) & 0 & (1-\eta) & 0 & (1+\eta) & 0 & -(1+\eta) \\ 0 & -(1-\xi) & 0 & -(1+\xi) & 0 & (1+\xi) & 0 & (1-\xi) \end{bmatrix}}^{[\mathbf{D}]}.$$

$$\{\boldsymbol{\varepsilon}\} = [\mathbf{A}][\mathbf{D}]\{\mathbf{U}\} \qquad \overbrace{\begin{Bmatrix} U_{ix} \\ U_{iy} \\ U_{jx} \\ U_{jy} \\ U_{mx} \\ U_{my} \\ U_{nx} \\ U_{ny} \end{Bmatrix}}^{\{\mathbf{U}\}} \qquad (8.84)$$

$$\Lambda^{(e)} = \frac{1}{2}(t_e)\int_A \{\boldsymbol{\varepsilon}\}^T[\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}\,dA = \frac{1}{2}(t_e)\int_{-1}^{1}\int_{-1}^{1}\{\boldsymbol{\varepsilon}\}^T[\boldsymbol{\nu}]\{\boldsymbol{\varepsilon}\}\overbrace{\det \mathbf{J}\,d\xi\,d\eta}^{dA}$$

$$\{\boldsymbol{\varepsilon}\} = [\mathbf{A}][\mathbf{D}]\{\mathbf{U}\}$$

$$[\mathbf{K}]^{(e)} = t_e\int_{-1}^{1}\int_{-1}^{1}[[\mathbf{A}][\mathbf{D}]]^T[\boldsymbol{\nu}][\mathbf{A}][\mathbf{D}]\det \mathbf{J}\,d\xi\,d\eta$$

More in general…

Analisi statica: le equazioni

- In static cases, the system of equilibrium equations has following form:

$$[K]\{D\}=\{F\}$$

- The displacement vector $\{D\}$ contains displacements of all degrees of freedom.
- The force vector $\{F\}$ contains forces acting on all degrees of freedom.
- The matrix $[K]$ is called the *stiffness matrix* of the structure. In a special case when the structure is a spring, $\{F\}$ as external force, and $\{D\}$ as the deformation of the spring, then $[K]$ is the spring constant.

La procedura di base del metodo agli elementi finiti

1. Given the bodies' geometries, material properties, support conditions, and loading conditions.
2. Divide the bodies into elements.
3. Establish the equilibrium equation: $[K]\{D\} = \{F\}$

   3.1 Construct the $[K]$ matrix, according to the elements' geometries and the material properties.

   3.2 Most of components in $\{F\}$ can be calculated, according to the loading conditions.

   3.3 Most of components in $\{D\}$ are unknown. Some component, however, are known, according to the support conditions.

   3.4 The total number of unknowns in $\{D\}$ and $\{F\}$ should be equal to the total number of degrees of freedom of the structure.

4. Solve the equilibrium equation. Now, the nodal displacements {d} of each element are known.
5. For each element:

   5.1 Calculate displacement fields $\{u\}$, using an interpolating method, $\{u\} = [N]\{d\}$. The interpolating functions in $[N]$ are called the *shape functions*.

   5.2 Calculate strain fields according to the strain-displacement relations.

   5.3 Calculate stress fields according to the stress-strain relations (Hooke's law).

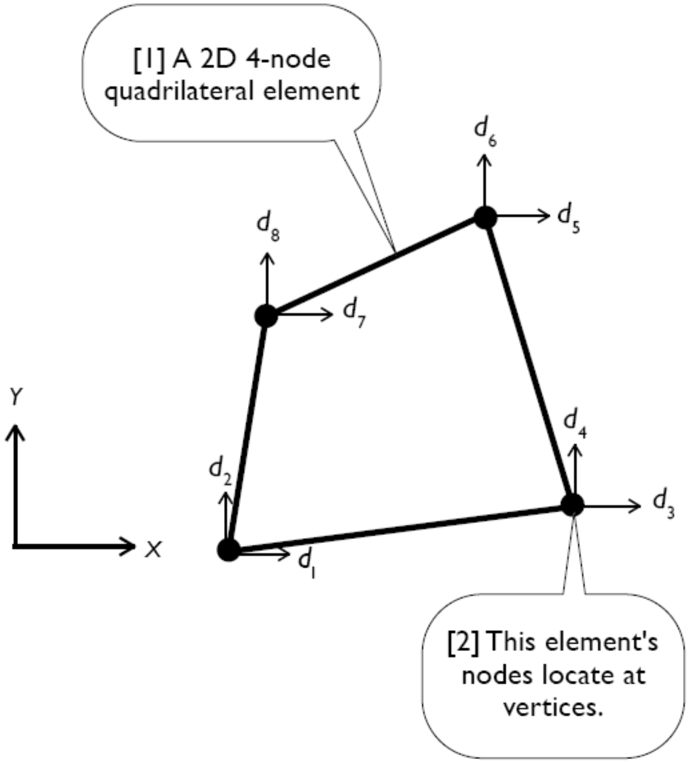
Le shape functions

-

- Shape functions serve as interpolating functions, allowing the calculation of displacement fields (functions of X, Y, Z) from nodal displacements (discrete values).

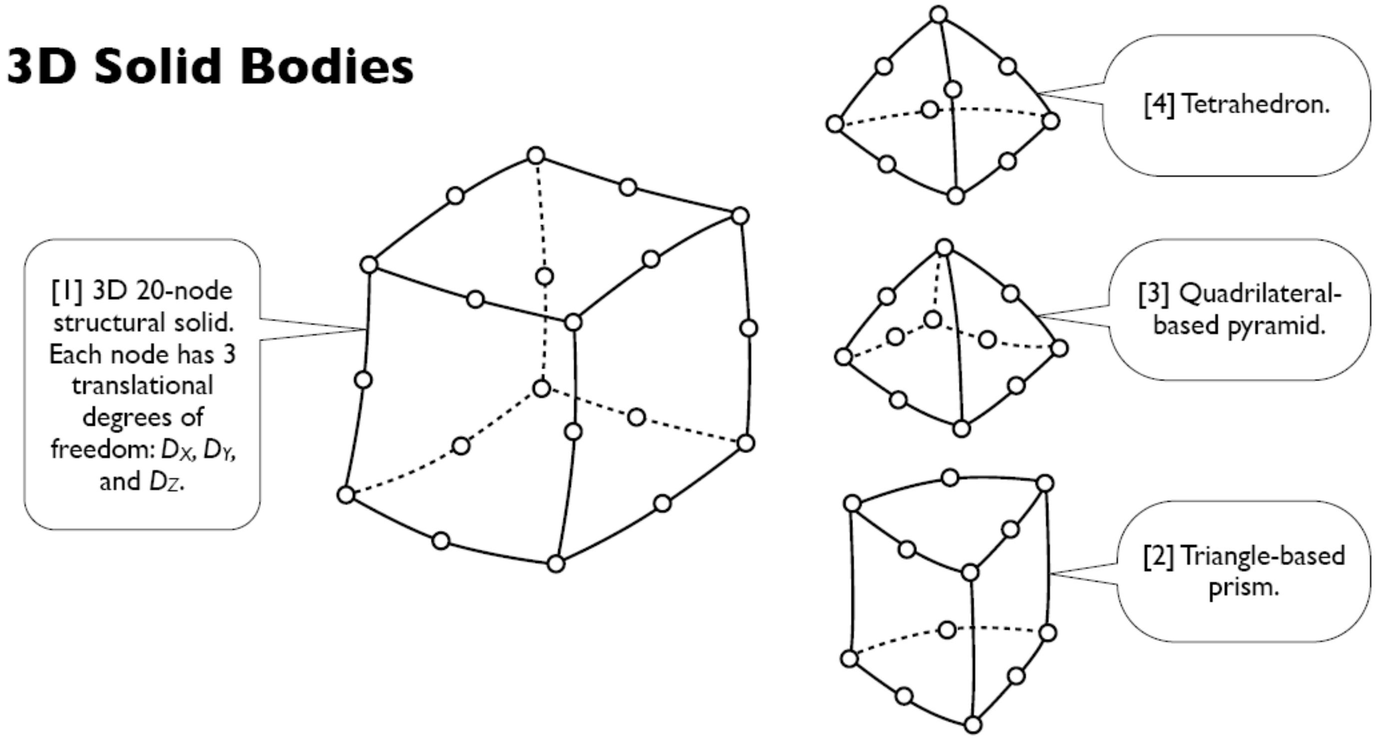
$$\{u\} = [N]\{d\}$$

- For elements with nodes at vertices, the interpolation must be linear and thus the shape functions are linear (of X, Y, Z).

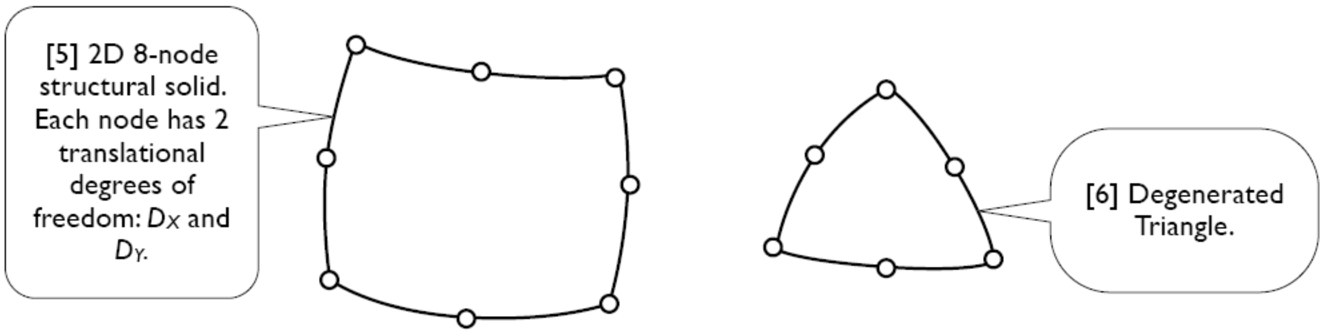


- For elements with nodes at vertices as well as at middles of edges, the interpolation must be quadratic and thus the shape functions are quadratic (of X, Y, Z).
- Elements with linear shape functions are called *linear elements*, *first-order elements*, or *lower-order elements*.
- Elements with quadratic shape functions are called *quadratic elements*, *second-order elements*, or *higher-order elements*.
- ANSYS Workbench supports only first-order and second-order elements.

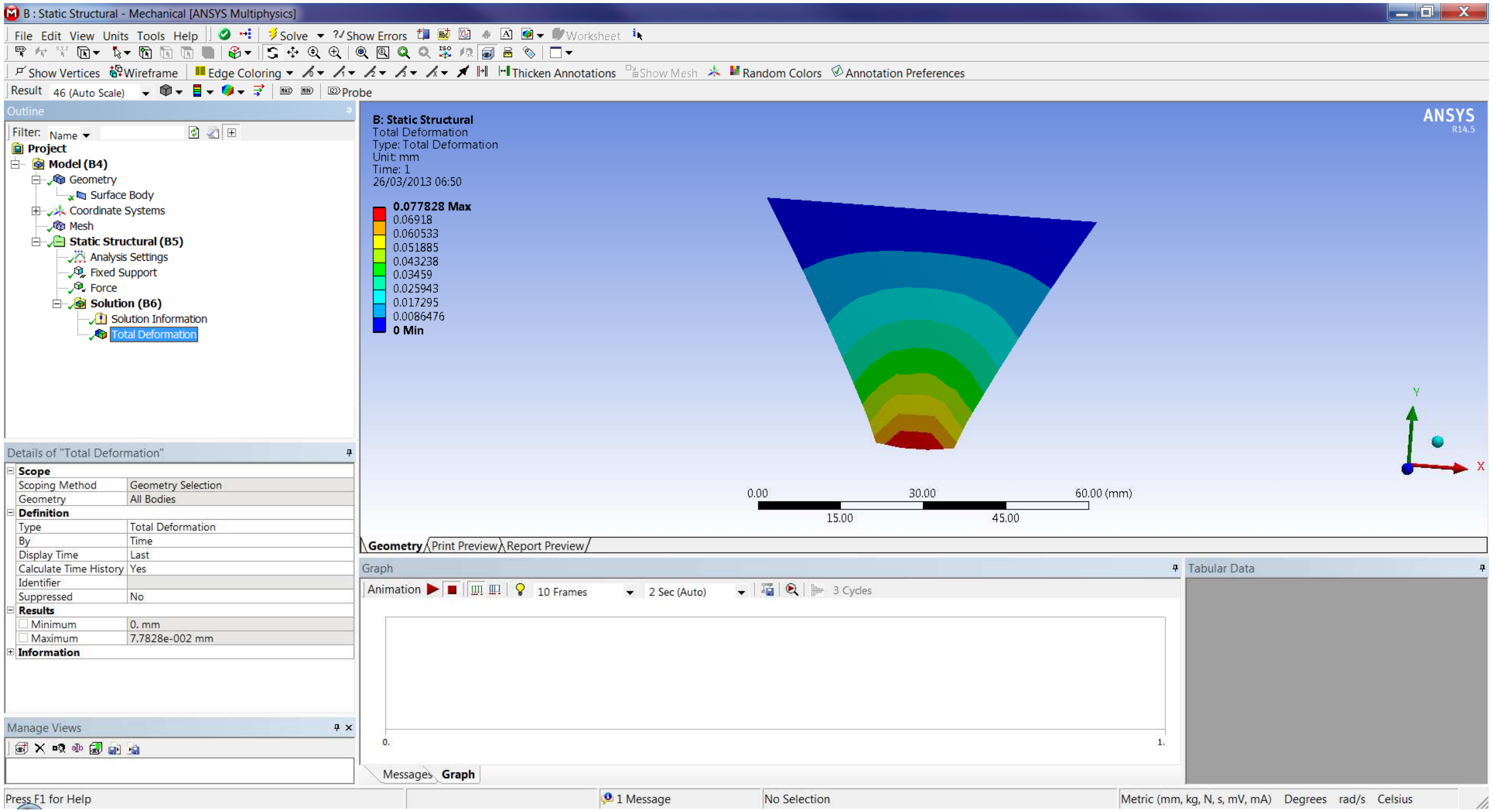
Esempio di elementi



2D Solid Bodies



And now let's see in workbench


B : Static Structural - Mechanical [ANSYS Multiphysics]
File Edit View Units Tools Help
Solve
Show Errors
Worksheet
Show Vertices
Wireframe
Edge Coloring
Thicken Annotations
Show Mesh
Random Colors
Annotation Preferences
Result 46 (Auto Scale)
Probe
Outline
Filter: Name
Project
Model (B4)
Geometry
Surface Body
Coordinate Systems
Mesh
Static Structural (B5)
Analysis Settings
Fixed Support
Force
Solution (B6)
Solution Information
Total Deformation
Details of "Total Deformation"
Scope
Scoping Method Geometry Selection
Geometry All Bodies
Definition
Type Total Deformation
By Time
Display Time Last
Calculate Time History Yes
Identifier
Suppressed No
Results
Minimum 0. mm
Maximum 7.7828e-002 mm
Information
Manage Views
B: Static Structural
Total Deformation
Type: Total Deformation
Unit: mm
Time: 1
26/03/2013 06:50
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0.06918
0.060533
0.051885
0.043238
0.03459
0.025943
0.017295
0.0086476
0 Min
ANSYS
R14.5
0.00
15.00
30.00
45.00
60.00 (mm)
Geometry
Print Preview
Report Preview
Graph
Animation
10 Frames
2 Sec (Auto)
3 Cycles
Tabular Data
Messages
Graph
Press F1 for Help
1 Message
No Selection
Metric (mm, kg, N, s, mV, mA) Degrees rad/s Celsius

thank you!

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PERCRO Perceptual Robotics Laboratory