ISTITUTO DI TECNOLOGIE DELLA COMUNICAZIONE, DELL'INFORMAZIONE E DELLA PERCEZIONE

PERCRO Perceptual Robotics Laboratory

Scuola Superiore Sant'Anna

Elementi di analisi strutturale

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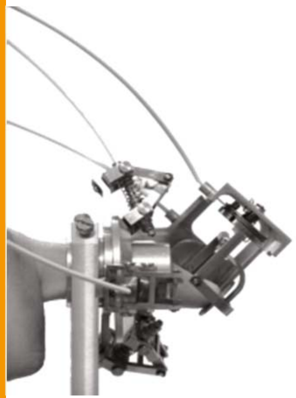
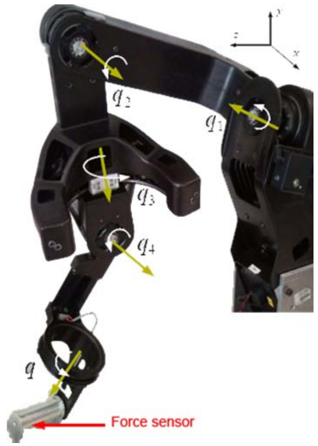
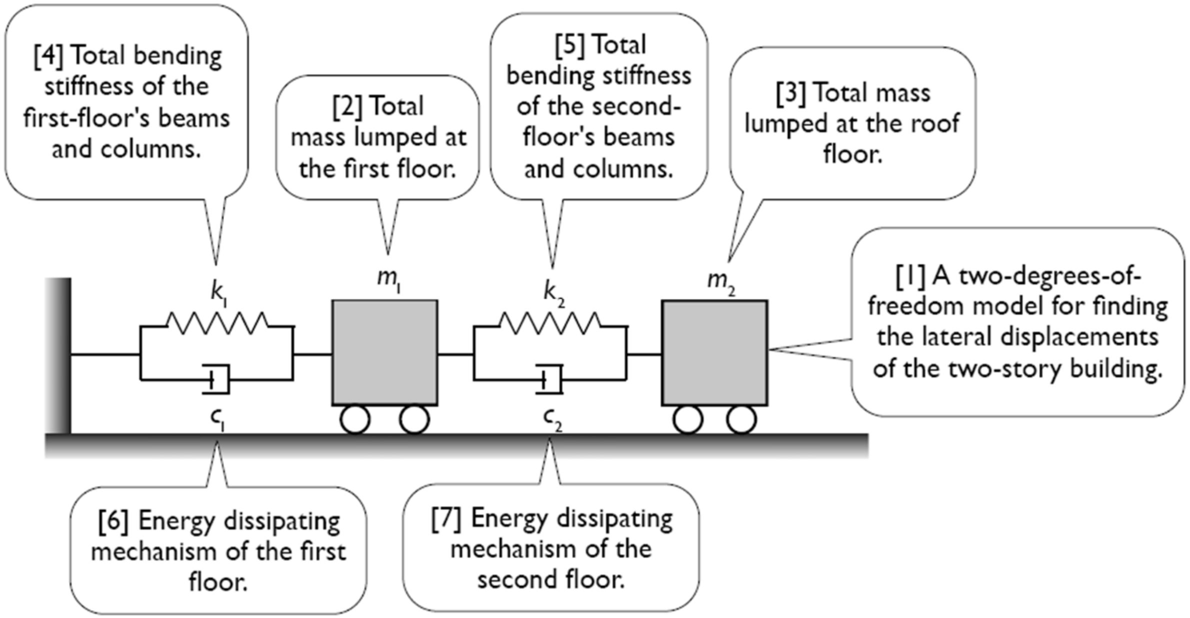




The 2-mass model

- The 2-mass model, theory behind

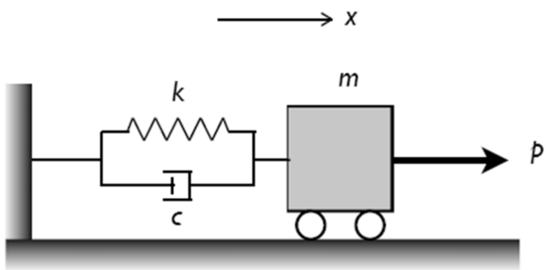


Single dof model

$$\sum F = ma$$

$$p - kx - c\dot{x} = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = p$$



Undamped vibrations

If no external forces exist, the equation for the one-degree-of-freedom system becomes

$$m\ddot{x} + c\dot{x} + kx = 0$$

If the damping is negligible, then the equation becomes

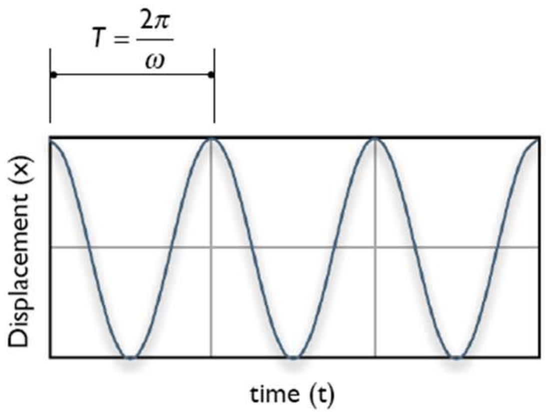
$$m\ddot{x} + kx = 0$$

The

$$x = A\sin(\omega t + B)$$

Natural frequency: $\omega = \sqrt{\frac{k}{m}}$ (rad/s) or $f = \frac{\omega}{2\pi}$ (Hz)

Natural period: $T = \frac{1}{f}$



Damped free vibrations

$$m\ddot{x} + c\dot{x} + kx = 0$$

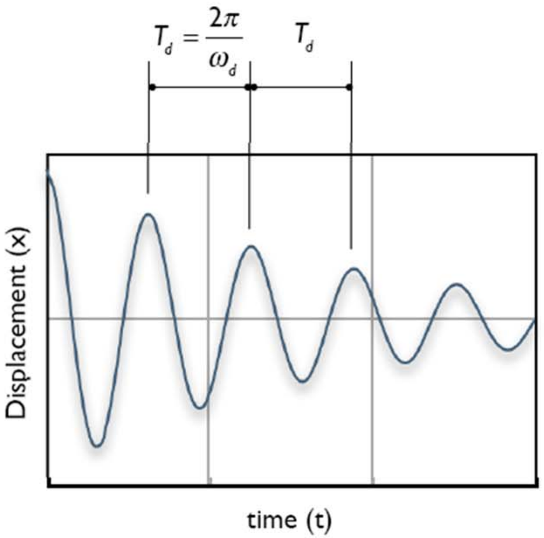
If the damping c is small (smaller than c_c), then the general solution is

$$x = Ae^{-\xi\omega t}\sin\left(\omega_d t + B\right)$$

Where

$$\omega_d = \omega\sqrt{1-\xi^2}\ ,\ \xi = \frac{c}{c_c}\ ,\ c_c = 2m\omega$$

The quantity c_c is called the *critical damping coefficient* and the quantity ξ is called the *damping ratio*.



Damping mechanisms in structures

- Damping is the collection of all energy dissipating mechanisms.
- In a structural system, all energy dissipating mechanisms come down to one word: friction. Three categories of frictions can be identified:
  - friction between the structure and its surrounding fluid, called *viscous damping*;
  - internal friction in the material, called *material damping*, *solid damping*, or *elastic hysteresis*;
  - friction in the connection between structural members, called *dry friction* or *Coulomb friction*.

Generalization to multiple degrees freedom

The foregoing concepts may be generalized to multiple degrees-of-freedom cases,

$$[M]\{\ddot{D}\}+[C]\{\dot{D}\}+[K]\{D\}=\{F\}$$

Where $\{D\}$ is the nodal displacements vector, $\{F\}$ is the nodal external forces vector, $[M]$ is called the *mass matrix*, $[C]$ is called the *damping matrix*, and $[K]$ is the stiffness matrix.

Note that when the dynamic effects (inertia effect and damping effect) are neglected, it reduces to a stati structural analysis system,

$$[K]\{D\}=\{F\}$$

Modal analysis

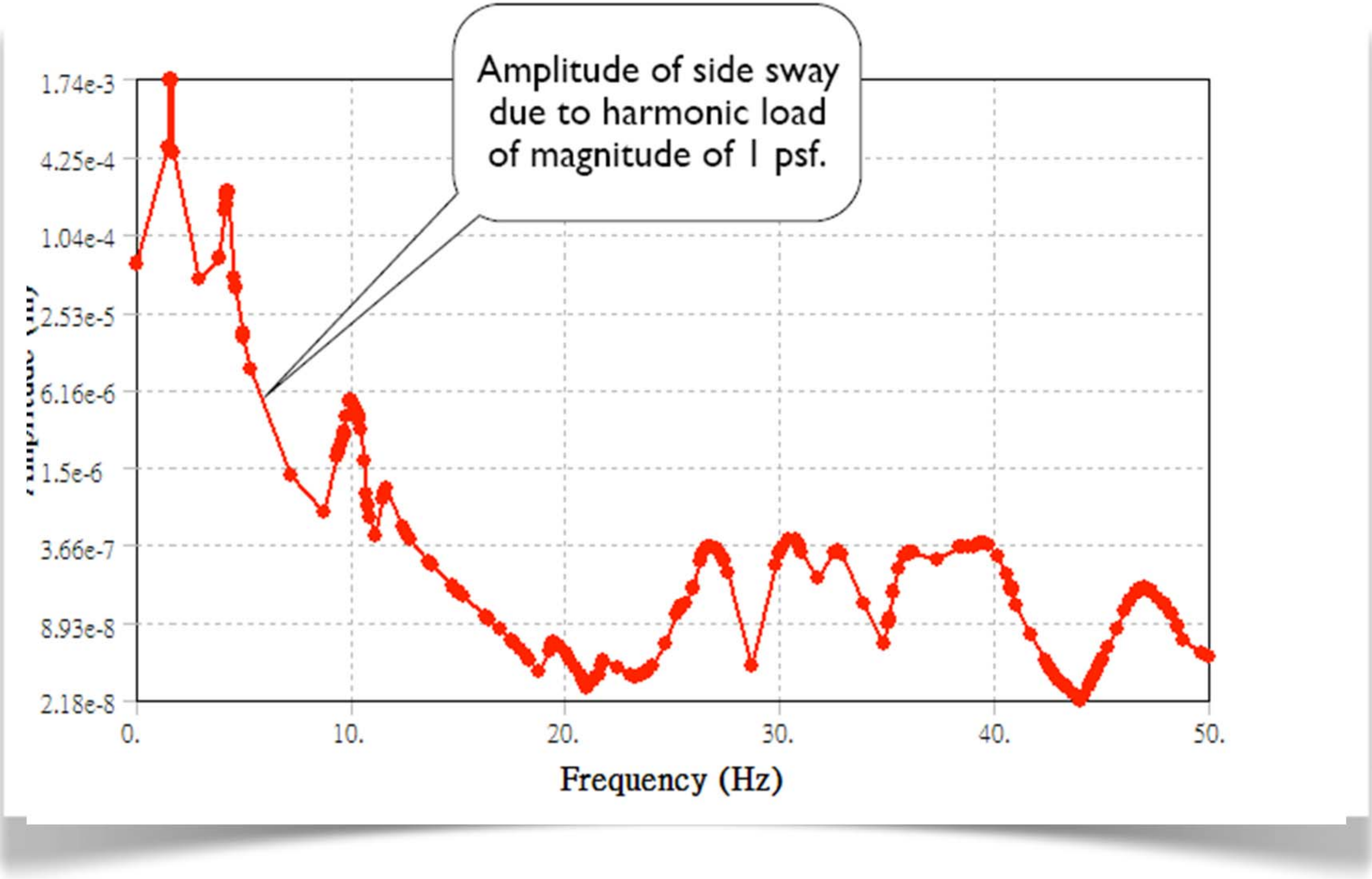
$$[M]\{\ddot{D}\}+[C]\{\dot{D}\}+[K]\{D\}=0$$

For a problem of n degrees of freedom, it has at most n solutions, denoted by $\{D_i\}, i=1,2,...,n$. These solutions are called *mode shapes* of the structure. Each mode shape $\{D_i\}$ can be excited by an external excitation of frequency ω_i, called the natural frequency of the mode.

In a modal analysis, since we are usually interested only in the natural frequencies and the shapes of the vibration modes, the damping effect is usually neglected to simplify the calculation,

$$[M]\{\ddot{D}\}+[K]\{D\}=0$$

Harmonic response analysis



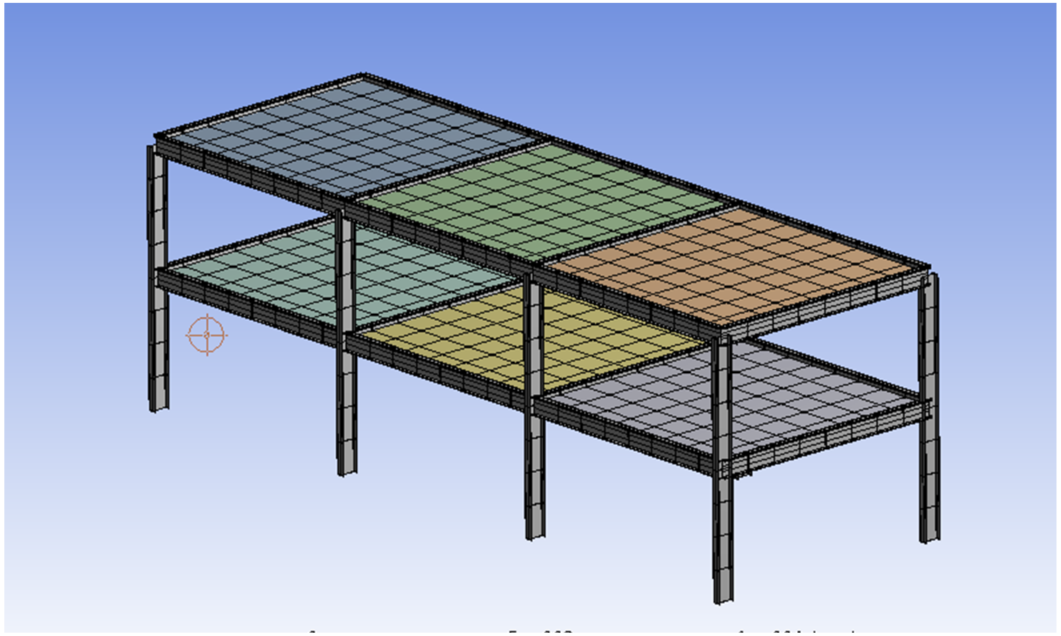
Transient structural analysis

$$[M]\{\ddot{D}\}+[C]\{\dot{D}\}+[K]\{D\}=\{F\}$$

<Transient Structural> analysis solves the general form of the equation. External force $\{F\}$ can be time-dependent forces. All nonlinearities can be included. It uses a *direct integration method* to calculate the dynamic response.

The direct integration method used in <Transient Structural> analysis is called an *implicit integration method*.

- The building is made of columns and bema modeled as line bodies and then floor slabs that are modeled as solid surfaces (reinforced concrete)
- The maximum allowed load is that of 50 lb/ft2 that is 50 psf (250 Kg m^2)

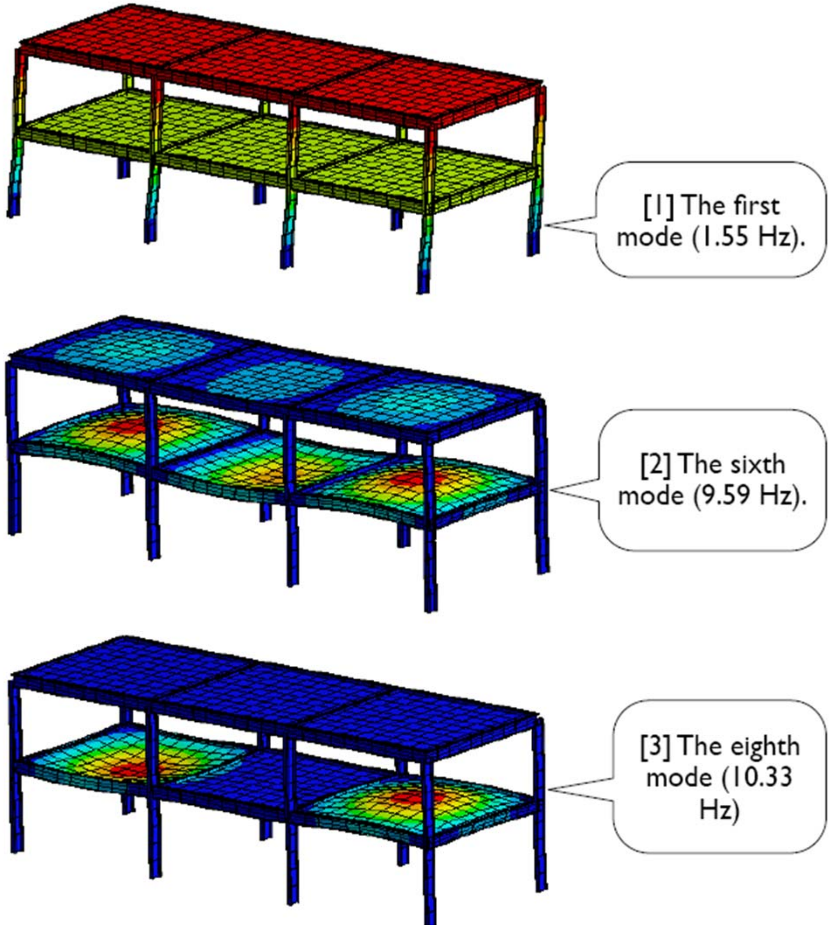


- The idea is to evalute the effects of two different kind of loads
  - o People dancing with oscillation of maximum load of 50 psf (250 Kg m^2) in the range 40 to 60 psf, so 10 psf (48 Kg/m^2, 0.00047 MPa)
    - Let's consider this issue for a frequency of about 1 Hz estimated
  - o The same analysis for a machine operating on the floor up to 3000 rpm (50 Hz). The machine weight is 40 lb (18.14 Kg), that due to the floor dimensions can be considered distributed as 0.1 psf (0.5 Kg/m2)

Building: modal analysis

Tabular Data

	Mode	Frequency [Hz]
1	1.	1.5543
2	2.	4.1017
3	3.	4.1821
4	4.	4.8874
5	5.	9.3546
6	6.	9.5862
7	7.	9.909
8	8.	10.327
9	9.	10.727
10	10.	11.495
11	11.	13.651
12	12.	16.327
13	13.	17.542
14	14.	18.235
15	15.	19.307
16	16.	20.092
17	17.	20.829
18	18.	21.174
19	19.	21.657
20	20.	23.192



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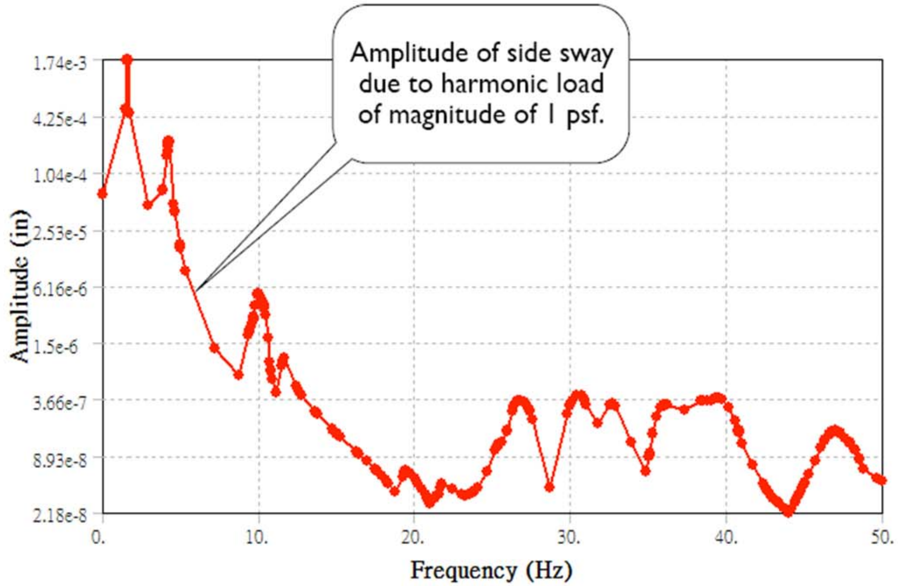
Example of influence of dancing on floor

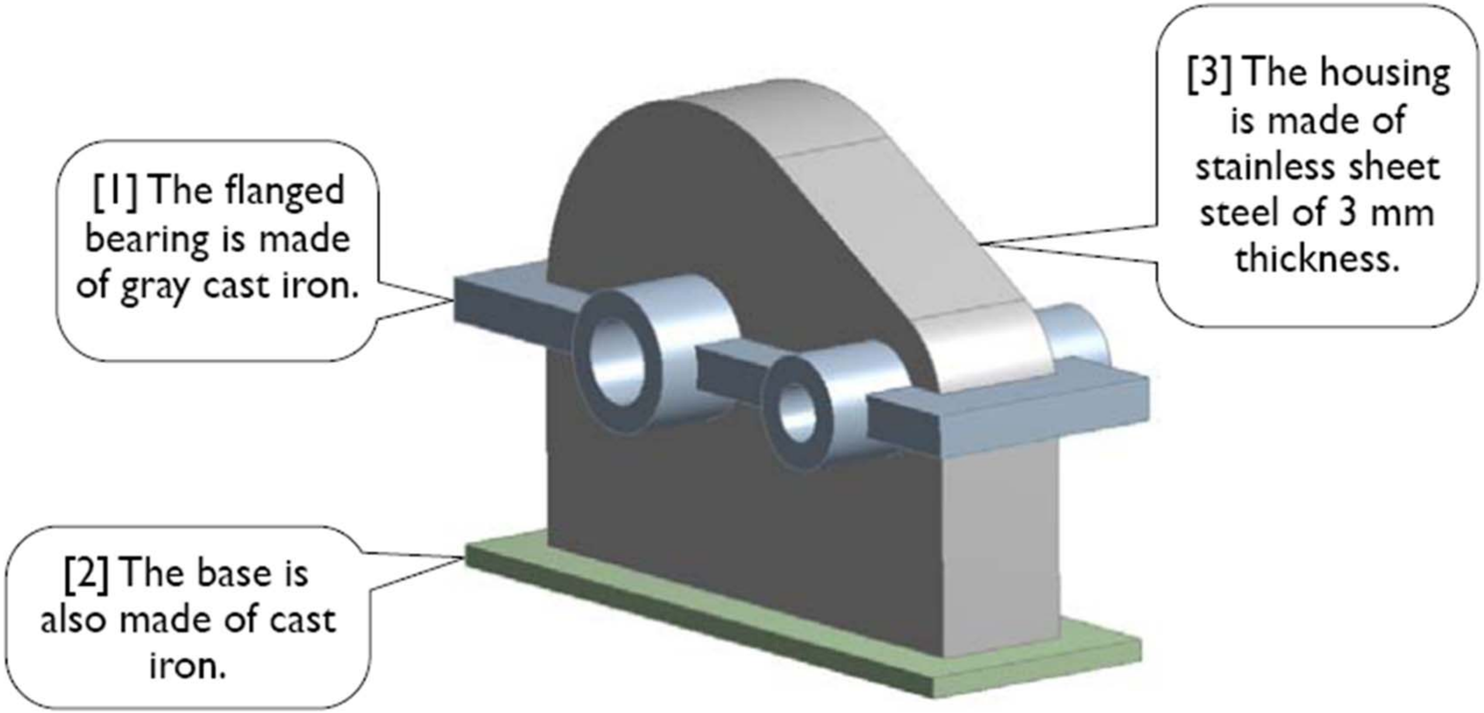
- The dancing frequency is close to the fundamental mode (1.55 Hz), that's why we pay attention to this mode, which is a side sway mode (in X-direction).

At dancing frequency of 1.55 Hz, the structure is excited such that the maximum X-displacement is 0.0174 in (0.44 mm). This value is too small to be worried about.

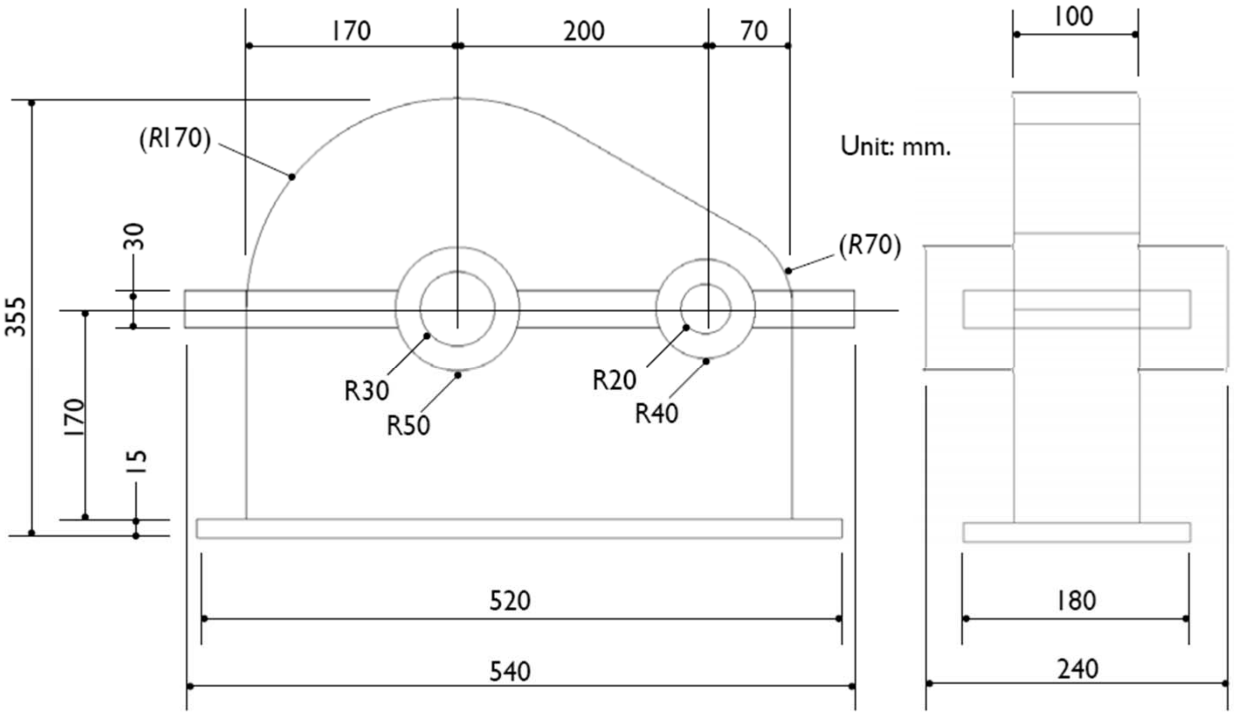


Analisi dinamica di un riduttore

- Nell'analisi di un riduttore le deformazioni devono essere tenute sotto controllo in quanto possono determinare uno spostamento degli assi del riduttore e quindi un cattivo funzionamento



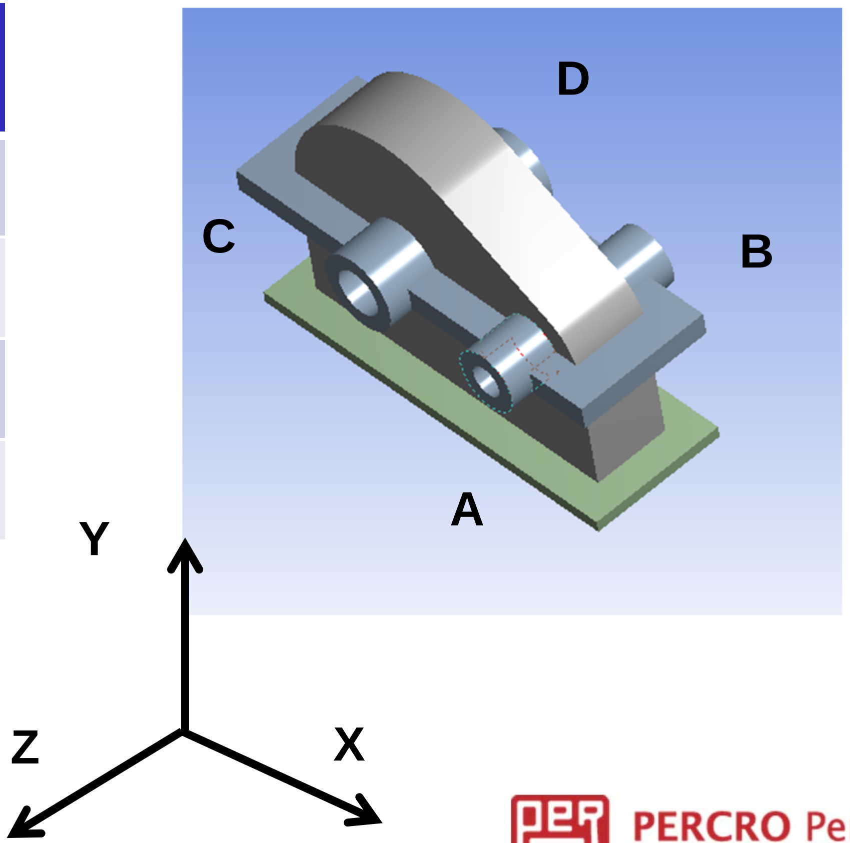
- Supponiamo di analizzare la scatola di un riduttore con le seguenti caratteristiche per una trasmissione di potenza



Carichi

- Supponiamo di aver calcolato i carichi associati ai cuscinetti del ns sistema. Cuscinetti A e D bloccati assialmente soltanto

Bearing	Fx (N) radiale	Fy (N) radiale	Fz(N) Assiale
A	16000	30000	16000
B	6000	30000	0
C	3000	-30000	0
D	-25000	-30000	-16000



- Modal analysis
  - Remember to transfer the constraint settings from previous static analysis
  - The connection can be implemented by simple transfering the static model in the modal box
  - Model shape results need to be created from the table by right click and select <create model shape results>

- To perform an harmonic response you should move the harmonic block on the modal one
- Be sure to transport again the fixed constraint
- For the analysis settings consider the following ones are possible valid ones

Details of "Analysis Settings"

Options	
Range Minimum	0. Hz
Range Maximum	50. Hz
Cluster Number	4
Solution Method	Mode Superposition
Cluster Results	Yes
Modal Frequency Range	Program Controlled
Store Results At All Frequencies	Yes
Output Controls	
Stress	No
Strain	No
Nodal Forces	No
Calculate Reactions	Yes
General Miscellaneous	No
Damping Controls	
Constant Damping Ratio	2.e-003
Stiffness Coefficient Define By	Direct Input
Stiffness Coefficient	0.

Manage Views

Vibration mode for a ball-screw actuator